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A game theory approach to the sawnwood and pulpwood markets in the north of Iran

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ABSTRACT
Duopoly game theory is applied to the wood industrial markets (sawnwood and pulpwood markets) in the North of Iran. The Nash equilibrium and the dynamic properties of the system based on marginal adjustments are determined. The probability that the Nash equilibrium will be reached is almost zero. The dynamical properties of sawnwood and pulpwood prices derived via the duopoly game model are found also in the real empirical price series.

Keywords: Sawnwood and pulpwood prices, dynamic duopoly game, Nash equilibrium, mixed strategy.

INTRODUCTION
The forest sector is important to the economy in northern Iran. Two sawmills dominate the sawnwood and pulpwood markets in the region. The analysis in this paper is made with the ambition to describe the sawnwood and pulpwood market structure and to analyze the dynamic properties of the market. The study focuses on the theory of dynamic duopoly games.

Game theory is the study of interacting decision makers. Game theory has been widely used in economics. Most economics behaviour can be viewed as a special case of game theory. A game model includes a set of players, a set of strategies and a set of payoffs that indicate the utility that each player receives if a particular combination of strategies is chosen. In the game model in this paper, there are two sawmills in the north of Iran that produce large amounts of sawnwood and pulpwood. Both mills are large enough to influence the market price. The study focuses on the theory of dynamic duopoly games. Each sawmill (player) has in the example two different possible strategies (decisions): A high (H) or a low (L) prices. Which price should you set? What is the optimal price of the other player?

How will decision makers (players), in a game change behaviour over time when they do not know how the different participants are affected by different possible actions taken by the other players? This general question will be addressed in this paper. The Nash equilibrium of this game will be determined. It will not be assumed that the players know exactly how the other players are affected by different decision combinations. Each player however observes the frequencies of the decisions taken by other player.

In the Nash equilibrium, no player has anything to gain by changing his own strategy (Nash 1950). Let (S, π) be a game, where S is the set of strategies and π is the set of payoffs. When each player $i \in [1, n]$ chooses strategy $x_i \in S_i$ resulting in strategy profile $x = (x_1, ..., x_n)$ then player i obtains payoff $\pi_i(x)$. A strategy profile $x^* \in S$ is a Nash equilibrium if no deviation in strategy by any single player is profitable. If the opponent of one player plays according to the Nash equilibrium, it is also profitable for the other to play the Nash equilibrium:

$$\pi_i(x_1^*, x_2^*, ..., x_{i-1}^*, x_i, x_{i+1}^*, ..., x_{n-1}^*, x_n^*) \leq \pi_i(x_1^*, x_2^*, ..., x_{i-1}^*, x_i^*, x_{i+1}^*, ..., x_{n-1}^*, x_n^*)$$

When the opponent does not play according to the Nash equilibrium, it may be profitable also for you to deviate from the Nash equilibrium:

$$\pi_i(x_1^*, x_2, ..., x_{i-1}^*, x_i, x_{i+1}^*, ..., x_{n-1}^*, x_n^*) \substack{<\\=\\>} \pi_i(x_1^*, x_2, ..., x_{i-1}^*, x_i^*, x_{i+1}^*, ..., x_{n-1}^*, x_n^*)$$

In this research we will deal with a duopoly game. A duopoly game examines the interactions of two firms in a market. Each firms optimal output and prices are affected by the decisions of the other. Different duopoly games are mentioned below:

1. Cournot duopoly

The Cournot duopoly was first analyzed by Cournot (1838). Each firm chooses its output simultaneously. In our example, if the sawmills produce a total of q units of sawnwood and pulpwood, the market price will be $p(q)$, where $p(q)$ is the inverse demand curve facing these two sawmills. If q_i is the production level for sawmill i, then the market price will be $p(q_1 + q_2)$, and the profits of sawmill i are $\pi_i = p(q_1 + q_2)q_i$. In this game the strategy of firm i is its choice of production level.

The dynamics of Cournot games have been studied by Flåm (1990, 1996, 1999 and 2002) and Flåm and Zaccour (1991). This paper is based on a similar approach, in the sense that the decisions are continuously adjusted. In this paper, however, the decisions concern mixed strategy frequencies, not output volumes and prices directly. Mixed strategy dynamics has earlier been investigated by Lohmander (1997).This research is rather similar to the earlier study by Lohmander (1997), but he did not use empirical data. Here the empirical data from the north of Iran was used to investigate the dynamic duopoly game.

In forestry Cournot games have also been studied by Kallio (2001) and Angelsen (2001). Kallio (2001) investigated the possibility of noncompetitive behavior of the buyers in the Finnish pulpwood market. He simulated the buyers' behavior under alternative competition structures (perfect competition, Cournot oligopsony, and monopsony) and compared the simulated equilibria with the observed behavior in the years 1988–1997. Angelsen (2001) studied possible strategic interactions between the state and local community in games of tropical forestland appropriation.

2. Bertrand duopoly**:**

The theory was developed by Bertrand (1883). Firms choose prices rather than quantities. If one firm announced its price and the other firm followed, both firms would finally reach a position that neither would like to deviate from. This position is as Nash equilibrium. This game can be compared to the famous Prisoners Dilemma game, described by Aubin (1979). If both players cooperate, they can charge the monopoly price and each receives a share of the monopoly profit.

3. Stackelberg Duopoly:

Another game was introduced by von Stackelberg (1934). In this model, firms choose quantities but one firm chooses before the other. At some points in the history of the USA automobile industry, i.e. General Motors has seemed to play such a leadership role (Gibbons 1992).In the present study, each player uses the latest and locally available frequency information in the decision process.

METHOD

The real sawnwood and pulpwood prices series from the year 1990 until 2004 were collected from the two sawmills in the north of Iran. Appendices B and C and Fig. 1 and 2 show these series. The differences between sawnwood and pulpwood prices in different mills are shown in Fig. 3 and 4.

These sawmills are called Shafarod and Neka Chub. Now, we denote sawmills Shafarod and Neka Chub, A and B, respectively.

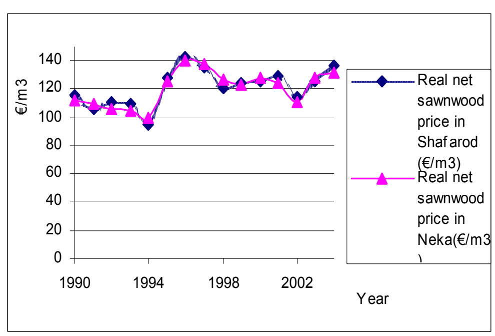

Fig 1. Real net sawnwood prices for two sawmills.

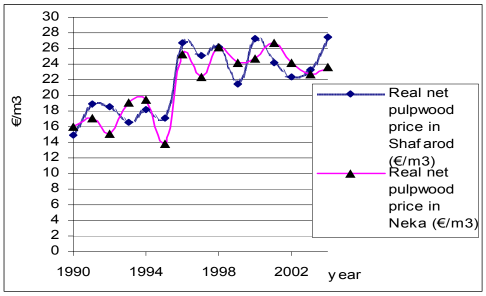

Fig 2. Real net pulpwood prices for two sawmills (The pulpwood is sold to pulp mills and other users).

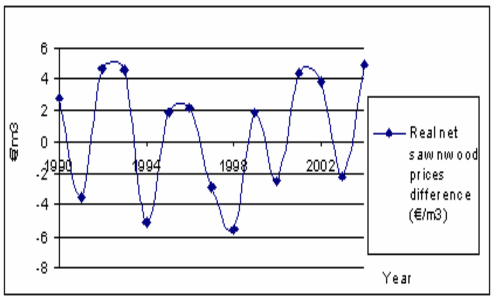

Fig.3. Sawnwood price differences between two sawmills.

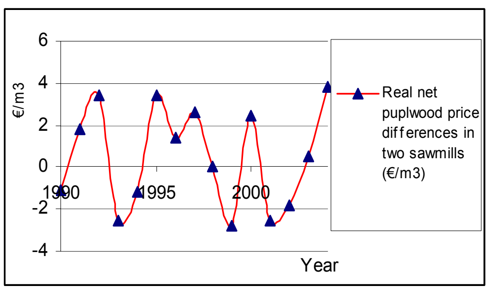

Fig.4. Pulpwood price differences between two sawmills.

Time series approach

To investigate the properties of the prices in two sawmills we start with a simple time series analysis. One may test many different kinds of time series models. We start with the most simple type, the first order autoregressive (AR) model.

1. Sawnwood price:

The sawnwood price data were used to estimate the following first order AR process:

$$P_{t+1} = \alpha + \beta \; P_t + \varepsilon_{\; t+1} \qquad (1)$$

where P_{t+1} is the expected price in period t+1, P_t is price in the current period and ε_{t+1} is the error term. ε_{t+1} is assumed to be independent over time, identically distributed and Gaussian, with expected value 0 and standard deviations $\delta^{SA}_{\varepsilon_{t+1}}, \delta^{SB}_{\varepsilon_{t+1}}, \delta^{PA}_{\varepsilon_{t+1}}, \delta^{PB}_{\varepsilon_{t+1}}$.

The estimated parameters α, β are found below: (t statistics in parentheses).

Sawmill A:

$$P^{SA}_{t+1} = 70.237 + 0.435\, P^{SA}_{t} + \varepsilon^{SA}_{t+1}, \delta^{SA}_{\varepsilon_{t+1}} = 12.070 \qquad (2)$$

Sawmill B:

$$P^{SB}_{t+1} = 50.741 + 0.592 P^{SB}_{t} + \varepsilon^{SB}_{t+1}, \; \delta^{SB}_{\varepsilon_{t+1}} = 10.684$$

(1.749) (2.463) (3)

2. Pulpwood prices:

Sawmill A:

$$P^{PA}_{t+1} = 11.331 + 0.514 P^{PA}_{t} + \varepsilon^{PA}_{t+1}, \; \delta^{PA}_{\varepsilon_{t+1}} = 3.446 \qquad (4)$$

Sawmill B:

$$P^{PB}_{t+1} = 10.930 + 0.511 P^{PB}_{t} + \varepsilon^{PB}_{t+1}, \; \delta^{PB}_{\varepsilon_{t+1}} = 3.652$$

(2.188) (2.210) (5)

The parameter estimates of the first order AR models give low t-values. The time series models do not explain anything. Why do we find the cycles in the price differences?

Let us to introduce a game model for these two sawmills in market supply competition.

Market competition for sawnwood and pulpwood

To study the dynamic game in the products market, we will start by defining the expected profits (payoffs) functions.

The profits in the mills may be calculated by the following function:

$$\pi = 0.7VP_S + 0.3VP_P - P_T 1.2\, V \qquad (6)$$

Where π is the profit, P_S is the net sawnwood price, Pp is the net pulpwood price, P_T is the timber price, and V is the sum of sawnwood and pulpwood ($V = V_S + V_P$). 1.2 m^3 of timber is transformed to 0.7 m^3 of sawnwood, 0.3 m^3 of pulpwood and 0.2 m^3 of waste. Sawmill A has higher capacity than sawmill B. They are both located close to the forest, about 500 km away from each other. Each sawmill uses a mixed strategy and determines a high or a low price. Compare Tables 1 and 2. We determine the elements of the payoff matrix this way:

1. Timber price:

The real timber price series from the year 1990 until 2004 were collected from the two sawmills (Fig 5.).

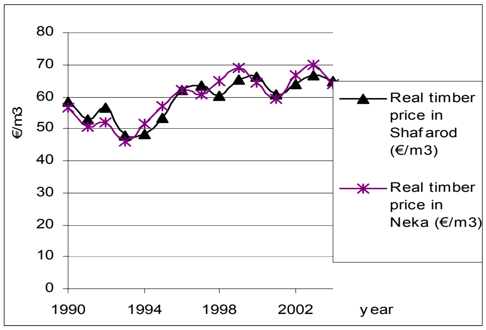

Fig. 5. Real timber prices in two sawmills in the north of Iran.

Note that "timber" is the raw material that becomes sawnwood and pulpwood in the sawmill. In some countries, timber is processed to sawnwood and chips.

To determine the expected payoff matrix we used different combinations of timber and products prices.

High timber price:

In case the sawnwood and pulpwood prices are high and the timber price is high (65 €/m3):

Ps = 125 (€/m^3), Pp = 25 (€/m^3), P_T = 65(€/m^3), V=1 m^3. If we substitute these values into Eq. 6, the profit is 17 €/ m^3.

In case the sawnwood and pulpwood prices are low and the timber price is high (65 €/m3):

Ps = 115 (€/m^3), Pp = 15 (€/m^3), P_T =65 (€/m^3), V=1 m^3.

By substituting these values into Eq. 6, the profit is 7 €/ m3.

In order to determine the Nash equilibrium (X,Y), the following functions were calculated:

E_A = 17.5XY + 21 X (1-Y) + 20.4Y (1-X) + 17(1-X) (1-Y) (7)

E_A= 17 +4X + 3.4Y – 6.9XY (8)

$\delta E_A/ \delta X$ = 4-6.9Y = 0 and Y = 0.579 (9)

Table 1: The payoffs matrix for two sawmills.

	Y	(1-Y)
X	V_A = 250 (1) V_B = 150 P_{sA} = 115 P_{SB} = 115 P_{pA} = 15 P_{PB} = 15 π_A= 17.50 (2) π_B= 10.50	V_A = 300 V_B = 50 P_{sA} = 115 P_{SB} = 125 P_{pA} = 15 P_{PB} = 25 π_A = 21.00 π_B = 8.50
(1-X)	V_A = 120 V_B = 200 P_{sA} = 125 P_{sB} = 115 P_{pA} = 25 P_{pB} = 15 π_A = 20.40 π_B = 14.00	V_A = 100 V_B = 90 P_{sA} = 125 P_{sB} = 125 P_{PA} = 25 P_{PB} = 25 π_A = 17.00 π_B = 15.30

X= Probability of low sawnwood and pulpwood prices of mill A.
(1-X)= Probability of high sawnwood and pulpwood prices of mill A.
Y= Probability of low sawnwood and pulpwood prices of mill B.
(1-Y)= Probability of high sawnwood and pulpwood prices of mill B.
1. V is the timber volume (1000 m^3).
2. π is the net profit (100000 €).

Expected payoff of mill B is:

E_B = 10.5XY +8.5 X (1-Y) + 14Y (1-X) + 15.3(1-X) (1-Y) (10)

E_B = 15.3 – 6.8X – 1.3Y + 3.3XY (11)

$\delta E_B/ \delta y$ = -1.3 + 3.3X = 0 and X = 0.394 (12)

The mixed Nash equilibrium is (N_X , N_Y) = (0.394, 0.579). The expected payoffs in Nash Equilibrium of sawmills A and B are 1897053 € and 1262091 €, respectively.

Low timber price:

In case the sawnwood and pulpwood prices are high and the timber price is low (55 €/m^3):

Ps = 125 (€/m^3), Pp = 25 (€/m^3), P_T = 55(€/m^3), V=1 m^3.

If we substitute these values into Eq. 6, the profit is 29 €/ m^3.

In case the sawnwood and pulpwood prices are low and the timber price is low (55 €/m^3):

Ps = 115 (€/m^3), Pp = 15 (€/m^3), P_T =55 (€/m^3), V=1 m^3.

By substituting these values into Eq. 6, the profit is 19 €/ m^3.

Expected payoff of mill A is:

E_A = 47.5XY + 57 X (1-Y) + 49.3Y (1-X) + 47.85(1-X) (1-Y) (13)

E_A= 47.85 + 9.15X +1.45Y – 10.95XY (14)

$\delta E_A/ \delta X$ = 9.15 – 10.95Y = 0 and Y = 0.836 (15)

Expected payoff of mill B is:

E_B = 38XY +29 X (1-Y) + 43.7Y (1-X) + 46.4(1-X) (1-Y) (16)

E_B = 46.6 – 17.4X – 2.7Y + 11.7XY (17)

$\delta E_B/ \delta Y$ = -2.7 + 11.7X = 0 and X = 0.231 (18)

The mixed Nash equilibrium is (N_X , N_Y) = (0.231, 0.836).

The expected payoffs in Nash equilibrium of sawmills A and B are 4906122 € and 4258285 €, respectively.

Table 2: The payoffs matrix for two sawmills.

	Low (Y)		High (1-Y)	
Low (X)	V_A = 250 (1)		V_A = 300	
	V_B = 200		V_B = 100	
	P_{sA} = 115	P_{SB} = 115	P_{sA} = 115	P_{SB} = 125
	P_{pA} = 15	P_{PB} = 15	P_{pA} = 15	P_{PB} = 25
	π_A= 47.50 (2)		π_A = 57.00	
	π_B= 38.00		π_B = 29.00	
High (1-X)	V_A = 170		V_A = 165	
	V_B = 230		V_B = 160	
	P_{sA} = 125	P_{sB} = 115	P_{sA} = 125	P_{sB} = 125
	P_{pA} = 25	P_{pB} = 15	P_{PA} = 25	P_{PB} = 25
	π_A = 49.30		π_A = 47.85	
	π_B = 43.70		π_B = 46.40	

X= Probability of low sawnwood and pulpwood prices of mill A.
(1-X)= Probability of high sawnwood and pulpwood prices of mill A.
Y= Probability of low sawnwood and pulpwood prices of mill B.
(1-Y)= Probability of high sawnwood and pulpwood prices of mill B.
1. V is the timber volume (1000 m^3).
2. π is the net profit (100000 €).

The dynamics of the mixed strategy game

The managers of the two mills do not have complete information concerning the properties of the other mills. The costs and revenues of the competitor are not perfectly known. The mixed strategy frequencies are however observed. Now, we introduce the dynamic rules of the game:

Each mill continuously observes the frequencies of the other mills action. The expected marginal profits, $\delta E_A/\delta X$ and $\delta E_B/\delta Y$ are calculated based on this information. In case the marginal profit of mill A is strictly positive (zero or strictly negative), mill A increases (leaves unchanged, decreases) X.

In case the marginal profit of mill B is strictly positive (zero or strictly negative), mill B increases (leaves unchanged, decreases) Y. We assume that the speed of adjustment (of X and Y) is proportional to the expected marginal profits and that both mills A and B have the same relation between speed of adjustment and expected marginal profit. We assume that W_1 and W_2 are the speed of adjustment for mills A and B, respectively and W_1=W_2.

1. The dynamics of the mixed strategy game when the timber price is high:

We may rewrite the Eq. (9) like:

$$\dot{X} = W_1 (\delta E_A/\delta X) \quad (19)$$

or $\dot{X} = W_1 (4-6.9Y)$ (20)

We can rewrite the Eq. (12) like,

$$\dot{Y} = W_2 (\delta E_B/\delta Y) \quad (21)$$

or $\dot{Y} = W_2 (-1.3 + 3.3X)$ (22)

The resulting mixed strategy trajectories are found in Fig. 4 when the timber price is high.

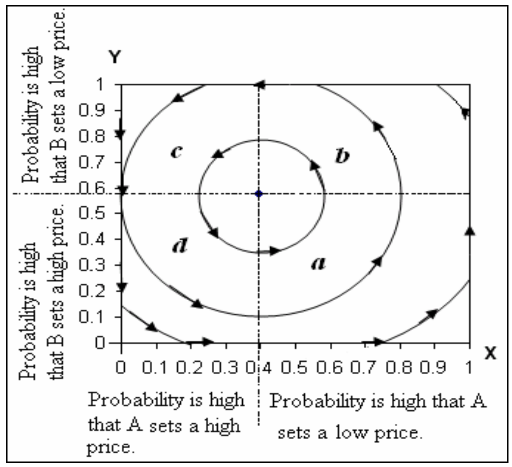

Fig. 4. The dynamics of the mixed strategies of the sawnwood and pulpwood game.

We can make the following observations in Fig. 4. The trajectories found in Fig.4 show possible time paths of the strategy combination (X, Y).

Region a:

X>0.394, Y<0.579. Sawmill A often sets low prices, and sawmill B often sets a high prices. Since A frequently sets a low prices, B finds it profitable to increase the frequency of low prices, so he decides to set low prices more often and the system moves upwards and to the right and soon reaches the region b.

Region b:

X>0.394, Y>0.579.Both mills often sets low prices. A realizes that it profitable if he increases the frequency of high prices, so he sets high prices more often and the system moves upwards and to the left, reaching region c.

Region c:

X<0.394, Y>0.579. Sawmill A often sets high prices, and sawmill B often sets low prices. B finds that it profitable to sets high price more often and the system moves down reaching region d.

Region d:

X< 0.394, Y<0.579. B prefers to frequently set high prices. A finds that it profitable if he more often sets low prices. He decides to increase the frequency of low prices and the system is moved to the right reaching region *a* again.

2. The dynamics of the mixed strategy game when the timber price is low:

We can write the dynamics of the mixed strategy sawnwood and pulpwood game when the timber price is low as:

We rewrite the Eq. (15) like:

$$\dot{X} = W_1(\delta E_A/\delta X) = 0 \qquad (23)$$

or

$$\dot{X} = W_1(9.15 - 10.95Y) \qquad (24)$$

We rewrite the Eq. (18) like,

$$\dot{Y} = W_2(\delta E_B/\delta Y) \qquad (25)$$

or

$$\dot{Y} = W_2(-2.7 + 11.7X) \qquad (26)$$

The resulting mixed strategy trajectories are found in Fig. 5 when the timber price is low.

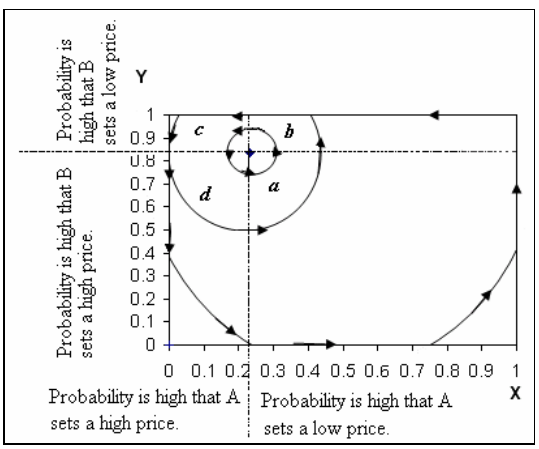

Fig. 5. The dynamics of the mixed strategies of the sawnwood and pulpwood game.

In Fig. 5, we can make the same observation as Fig. 4.

Formal analysis of the dynamics

The aim is to show that the expected payoffs (E_A and E_B) in the two sawmills follow the trajectories in Fig. 4 and 5.

The formal analysis of the differential equation is found in the Appendix A.

$$\dot{X} = \alpha_1 + \beta_1 Y \qquad (27)$$

$$\dot{Y} = \alpha_2 + \beta_2 X \qquad (28)$$

The following assumptions are satisfied:

$(\beta_1\beta_2 < 0)$, $(\alpha_1\beta_1 < 0)$, $(\alpha_2\beta_2 < 0)$

The solution is:

$$X(t) = A_1\cos(\theta t) + A_2\sin(\theta t) + N_X \qquad (29)$$

$$Y(t) = A_3\cos(\theta t) + A_4\sin(\theta t) + N_Y \qquad (30)$$

(N_X, N_Y) is the Nash equilibrium and

$$N_X = -\frac{\alpha_2}{\beta_2},\ N_Y = -\frac{\alpha_1}{\beta_1}.$$

$$X(0) = X_0,\ Y(0) = Y_0,\ A_1 = X_0 + \frac{\alpha_2}{\beta_2},\ A_2 = \frac{\beta_1 A_3}{\theta},$$

$$A_3 = Y_0 + \frac{\alpha_1}{\beta_1},\ A_4 = \frac{\beta_2 A_1}{\theta},\ \sqrt{-\beta_1\beta_2} = \theta.$$

The trajectories X(t) and Y(t) when timber price is high are shown in Fig. 6 and 7.

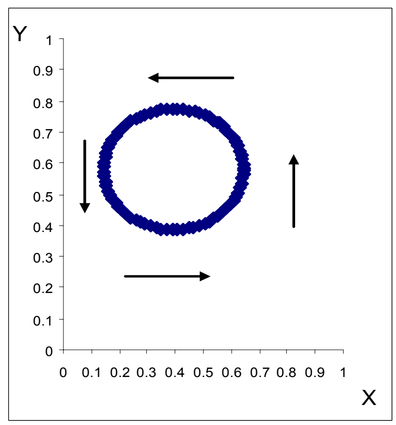

Fig. 6. The dynamics of the mixed strategies of the sawnwood and pulpwood game when the timber price is high for the two players A and B.

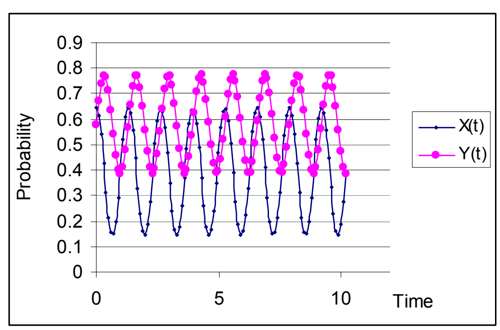

Fig. 7. The probability path of the mixed strategy sawnwood and pulpwood game when the timber price is high for the two players A and B.

The trajectories X(t) and Y(t) when the timber price is low are shown in Fig. 8 and 9.

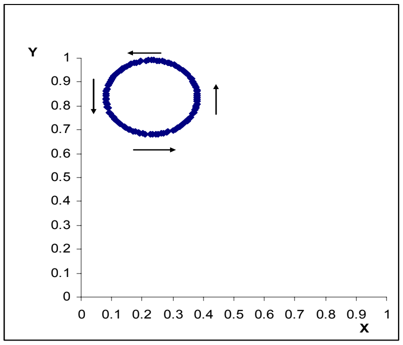

Fig. 8. The dynamics of the mixed strategies of the sawnwood and pulpwood game when the timber price is low for the two players A and B.

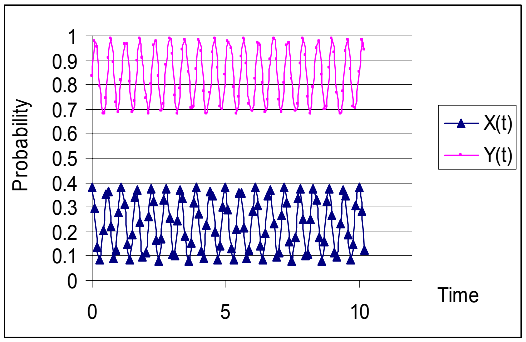

Fig. 9. The probability path of the mixed strategy sawnwood and pulpwood game when the timber price is low for the two players A and B.

(X(t), Y(t)) will follow an orbit around the Nash equilibrium (N_X, N_Y). This is called a center in the theory of dynamical systems.

CONCLUSION

This paper contains a simple treatment of the dynamics of two person non- zero-sum game in the sawnwood and pulpwood markets in the northern part of Iran where the players make use of local information and continuous decision frequency observations. The trajectories of the decision probability combinations were investigated. It was found that a large number of initial conditions make the decision probability combination follow a special form of attractor and that centers can be expected to appear in typical games. The probability that the Nash equilibrium will be reached is almost zero.The differential equation system governing the simultaneous optimal adjustments of the decision frequencies of the two players give cyclical solutions.

What is the advice to the players in real market games?

Observe the behaviour of the other players and optimize your expected profit conditional on that information. This seems to be a very simple and natural advice. One consequence is however that the market solution will be periodic. A stable equilibrium will generally not be found.

APPENDICES

Appendix C: Formal analysis of the dynamics:

$\dot{X} = \alpha_1 + \beta_1 Y$ (1)

$\dot{Y} = \alpha_2 + \beta_2 X$ (2)

We assume $(\beta_1\beta_2 < 0)$, $(\alpha_1\beta_1 < 0)$, $(\alpha_2\beta_2 < 0)$

$\ddot{X} = \beta_1 \dot{Y}$

$\ddot{X} = \beta_1(\alpha_2 + \beta_2 X)$

$\ddot{X} = \beta_1\alpha_2 + \beta_1\beta_2 X$ and $\ddot{X} - \beta_1\beta_2 X = \beta_1\alpha_2$ (3)

In general form we have

$\ddot{X}$ + aX-b = 0 where a = - $\beta_1\beta_2$ and b= $\beta_1\alpha_2$

Homogenous solution of equation (3):

$\dot{X} + aX = 0$ (4)

Let X(t)=Ae^{Lt}

$\dot{X}$ =LAe^{Lt} and $\ddot{X}$ = L^2Ae^{Lt} , Ae^{Lt} (L^2 + a)=0

L=$\pm\sqrt{\beta_1\beta_2}$ assume i= $\sqrt{-1}$

then L=$\pm\sqrt{-\beta_1\beta_2}$ i (5)

Particular solution of equation (3):

X(t)=m+nt.

$\dot{X}$ =n and $\ddot{X}$ =0 By using this results into equation (4), we get:

0+a (m+nt)=b

n_1=0 then am_1=b and m=$\frac{b}{a}$ so we get

$m_1=\frac{+\beta_1\alpha_2}{-\beta_1\beta_2}$ or $m_1=-\frac{\alpha_2}{\beta_2}$

As a consequence, we have X(t)= $Ae^{\pm\sqrt{-\beta_1\beta_2}\, it} + (-\frac{\alpha_2}{\beta_2})$

Hence,

X(t) = e^{0t}(A_1 cos ($\sqrt{-\beta_1\beta_2}$ t) + A_2 sin ($\sqrt{-\beta_1\beta_2}$ t) $-\frac{\alpha_2}{\beta_2}$

or X(t) = A_1 cos ($\sqrt{-\beta_1\beta_2}$ t) + A_2 sin($\sqrt{-\beta_1\beta_2}$ t) $-\frac{\alpha_2}{\beta_2}$ (6)

$\ddot{Y} = \beta_2\dot{X}$ (7)

By substituting equation (2) in equation (7) we get:

$$\ddot{Y} = \beta_2(\alpha_1 + \beta_1 Y)$$

$$\ddot{Y} = \beta_2\alpha_1 + \beta_1\beta_2 Y$$

$$\ddot{Y} - \beta_1\beta_2 Y = \beta_2\alpha_1$$

Finally we get this solution:

$$Y(t) = A_3 \cos(\sqrt{-\beta_1\beta_2}\ t) + A_4 \sin(\sqrt{-\beta_1\beta_2}\ t) - \frac{\alpha_1}{\beta 1} \qquad (8)$$

We assume $\sqrt{-\beta_1\beta_2} = \theta$ (9)

We rewrite equations (6) and (8) like this:

$$X(t) = A_1\cos(\theta\ t) + A_2\sin(\theta\ t) - \frac{\alpha_2}{\beta_2} \qquad (10)$$

$$Y(t) = A_3\cos(\theta\ t) + A_4\sin(\theta\ t) - \frac{\alpha_1}{\beta 1} \qquad (11)$$

The first order derivatives of these equations are:

$$\dot{X} = -A_1\theta\sin(\theta\ t) + A_2\theta\cos(\theta\ t) \qquad (12)$$

$$\dot{Y} = -A_3\theta\sin(\theta\ t) + A_4\theta\cos(\theta\ t) \qquad (13)$$

If we substitute equations (10) and (11) in equations (1) and (2), we have:

$$\dot{X} = \alpha_1 + \beta_1(A_3\cos(\theta\ t) + A_4\sin(\theta\ t) - \frac{\alpha_1}{\beta 1}$$

(14)

$$\dot{Y} = \alpha_2 + \beta_2(A_1\cos(\theta\ t) + A_2\sin(\theta\ t) - \frac{\alpha_2}{\beta_2})$$

(15)

After simplifying, we get:

$$\dot{X} = \beta_1 A_3\cos(\theta\ t) + \beta_1 A_4\sin(\theta\ t) \qquad (16)$$

$$\dot{Y} = \beta_2 A_1\cos(\theta t) + \beta_2 A_2\sin(\theta\ t) \qquad (17)$$

From equations (12, 13) and (16, 17) we may have the following equalities:

$$\begin{cases} \begin{cases} -A_1\theta = \beta_1 A_4 \\ A_2\theta = \beta_1 A_3 \end{cases} \\ \begin{cases} -A_3\theta = \beta_2 A_2 \\ A_4\theta = \beta_2 A_1 \end{cases} \end{cases} \qquad (18)$$

From equation (18), we get:

$$\frac{A_1}{A_2} = -\frac{A_4}{A_3}, \quad \frac{A_3}{A_4} = -\frac{A_2}{A_1}, \quad A_2 = \frac{\beta_1 A_3}{\theta},$$

$$A_3 = \frac{A_2\theta}{\beta_1}, \ A_4 = \frac{\beta_2 A_1}{\theta} \qquad (19)$$

Consequently, the following equations can be written:

$$\begin{cases} X(t) = A_1\cos(\theta t) + A_2\sin(\theta t) - \frac{\alpha_2}{\beta_2} \\ Y(t) = (-\frac{\beta_2 A_2}{\theta})\cos(\theta t) + (\frac{\beta_2 A_1}{\theta})\sin(\theta t) - \frac{\alpha_1}{\beta_1} \end{cases}$$

(20)

or

$$\begin{cases} X(t) = A_1\cos(\theta t) + (\frac{\beta_1 A_3}{\theta})\sin(\theta t) - \frac{\alpha_2}{\beta_2} \\ Y(t) = A_3\cos(\theta t) + (\frac{\beta_2 A_1}{\theta})\sin(\theta t) - \frac{\alpha_1}{\beta_1} \end{cases}$$

(21)

Then: $X(0) = A_1 - \frac{\alpha_2}{\beta_2} \Rightarrow A_1 = X(0) + \frac{\alpha_2}{\beta_2}$

$Y(0) = A_3 - \frac{\alpha_1}{\beta_1} \Rightarrow A_3 = Y(0) + \frac{\alpha_1}{\beta_1}$

The Nash equilibrium values for X and Y are $N_X = -\frac{\alpha_2}{\beta_2}$, $N_Y = -\frac{\alpha_1}{\beta_1}$, respectively.

Appendix B: Real net sawnwood price in two sawmills in the north of Iran.

Year	Real sawnwood price in Shafarod (€/m³)	Real sawnwood price in Neka Chub (€/m³)	Real sawnwood production cost in Shafarod (€/m³)	Real sawnwood production cost in Neka Chub (€/m³)	Real net sawnwood price in Shafarod (€/m³)	Real net sawnwood price in Neka Chub(€/m³)
1990	129.16	127.74	14.19	15.61	114.97	112.13
1991	119.04	123.75	12.96	14.14	106.07	109.61
1992	123.01	118.28	12.30	12.30	110.71	105.98
1993	120.84	120.07	11.55	15.39	109.29	104.68

Appendix B. Continued.

1994	108.34	114.04	13.68	14.25	94.65	99.78
1995	141.16	141.16	13.35	15.26	127.80	125.90
1996	158.03	158.03	15.49	17.66	142.54	140.37
1997	150.48	153.12	15.84	15.58	134.64	137.54
1998	138.59	143.07	17.88	16.77	120.71	126.30
1999	143.36	139.63	18.62	16.76	124.74	122.88
2000	142.17	145.47	17.36	18.18	124.81	127.29
2001	146.91	143.95	17.81	19.29	129.11	124.65
2002	133.28	128.16	19.22	17.94	114.06	110.21
2003	142.97	146.30	17.73	18.84	125.24	127.46
2004	153.00	148.00	17.00	17.00	136.00	131.00

Appendix C: Real net pulpwood price in two sawmills in the north of Iran.

Year	Real pulpwood price in Shafarod (€/m³)	Real pulpwood price in Neka Chub (€/m³)	Real pulpwood production cost in Shafarod (€/m³)	Real pulpwood production cost in Neka Chub (€/m³)	Real net pulpwood price in Shafarod (€/m³)	Real net pulpwood price in Neka Chub (€/m³)
1990	19.87	21.29	4.97	5.25	14.90	16.04
1991	23.57	22.39	4.71	5.30	18.86	17.09
1992	23.66	19.87	5.11	4.73	18.55	15.14
1993	20.78	23.09	4.23	4.00	16.55	19.09
1994	22.81	23.95	4.56	4.56	18.25	19.39
1995	20.98	17.17	3.82	3.43	17.17	13.73
1996	30.99	29.44	4.34	4.18	26.65	25.25
1997	29.04	26.40	3.96	3.96	25.08	22.44
1998	30.62	31.30	4.47	5.14	26.15	26.15
1999	26.06	28.86	4.65	4.65	21.41	24.20
2000	33.06	31.41	5.79	6.61	27.28	24.80
2001	31.16	34.13	6.97	7.42	24.19	26.71
2002	29.48	30.76	7.05	6.54	22.43	24.22
2003	29.92	28.82	6.65	6.10	23.27	22.72
2004	34.00	30.00	6.50	6.30	27.50	23.70

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